MTH 516/616: Topology II

Homework III

(Due 07/04)

1. Deduce that every continuous map $f : \mathbb{R}P^{2n} \to \mathbb{R}P^{2n}$ has a fixed point.

2. Construct a surjective map $f : S^n \to S^n$ of degree 0.

3. For an invertible linear transformation $f : \mathbb{R}^m \to \mathbb{R}^n$ show that the induced map on $H_n(\mathbb{R}^n, \mathbb{R}^n \setminus \{0\})$ is either id or $-id$ according as Det$f$ is positive or negative.

4. (a) Show that the quotient map $S^1 \times S^1 \to S^2$ obtained by collapsing the subspace $S^1 \vee S^1 \subset S^1 \times S^1$ is not nullhomotopic. [Hint: Consider the induces map on H_2.]

   (b) Show that every map $S^2 \to S^1 \times S^1$ is nullhomotopic. [Hint: Use covering space theory.]

5. Let X be the quotient space of S^2 obtained by identifying pairs of antipodal points on its equator. Compute its homology groups.

6. Using the Mayer-Vietoris sequence or otherwise, establish the following isomorphisms.

   (a) $H_i(X \times S^n) \cong H_i(X) \oplus H_i(X \times S^n, X \times \{x_0\})$.

   (b) $H_i(X \times S^n, X \times \{x_0\}) \cong H_{i-1}(X \times S^{n-1}, X \times \{x_0\})$.

7. Using the Mayer-Vietoris sequence, compute the homology groups of the 3-manifold obtained by gluing 2 copies of S_g using the identity map.

8. **(For practice)**

   (i) Problem 17, Section 2.1 (Page 132, A. Hatcher).

   (ii) Problem 29, Section 2.1 (Page 133, A. Hatcher).

   (iii) Problem 43, Section 2.2 (Page 159, A. Hatcher).

9. **Reading assignment**: Sections 2.B and 2.C.